Tutorial Physics 1 Week 3 Solutions

James Paynter

March 2021

The monkey on a rope

••59 SSM A 10 kg monkey climbs up a massless rope that runs over a frictionless tree limb and back down to a 15 kg package on the ground (Fig. 5-54). (a) What is the magnitude of the least acceleration the monkey must have if it is to lift the package off the ground? If, after the package has been lifted, the monkey stops its climb and holds onto the rope, what are the (b) magnitude and (c) direction of the monkey's acceleration and (d) the tension in the rope?

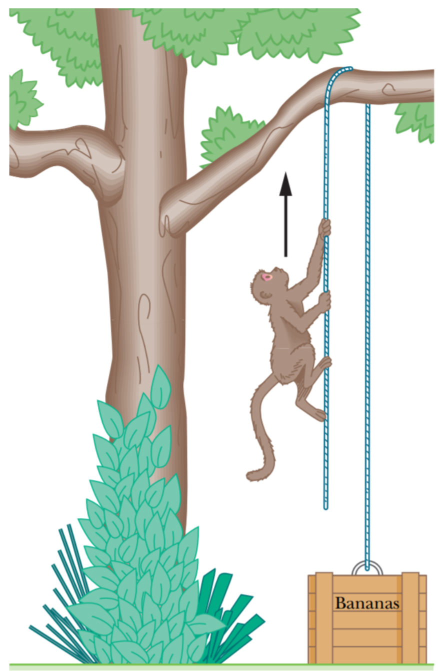

Fig. 5-54 Problem 59.

Figure 1: monkey mass, m_1, package mass, m_2

a)

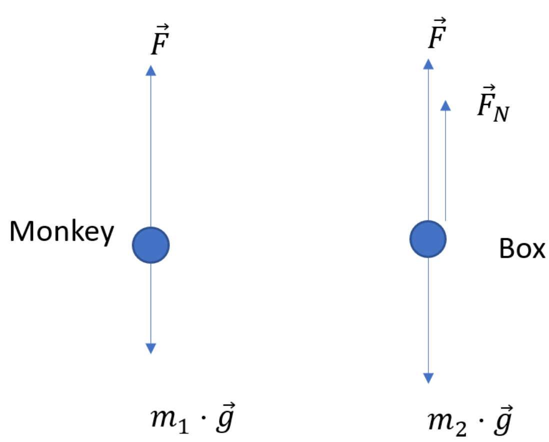

Figure 2: Free-body diagrams for monkey on a rope problem

For the monkey, the forces are

$$\vec{F}_{\text{net}} = m_1 \cdot \vec{a}_1 = \vec{F} - m_1 \cdot \vec{g}. \tag{1}$$

Since the rope is massless, the tension just becomes $F, \rightarrow T = F$.
For the box, we have

$$\vec{F}_{\text{net}} = m_2 \cdot \vec{a}_2 = \vec{F} + \vec{F}_N - m_2 \cdot \vec{g}.$$
$$\implies \vec{F} = m_2 \cdot \vec{g} \text{ (see following sentence).} \tag{2}$$

Which is the total force in the rope pulling down, $\vec{F}$, the normal force of the ground on the box, $\vec{F}_N$. $\vec{F}_N$ is zero at the point where the package *just* starts to lift, with $\vec{a}_2 = 0$.

So, the equation for the net force on the monkey is (by subbing (2) into (1)):

$$\begin{aligned}\vec{F}_{\text{net}} &= m_1 \cdot \vec{a}_1 = \vec{F} - m_1 \cdot \vec{g} \\ \vec{a}_1 &= \frac{m_2 \cdot \vec{g} - m_1 \cdot \vec{g}}{m_1} \\ &= \frac{(15-10)\cdot 10}{10} = 5 \text{ m sec}^{-2}\end{aligned}$$

The key point here is that the monkey and the box have different accelerations, so they cannot be considered part of the same system (In terms of Newton's second law, i.e. $\vec{F} = m\vec{a}$).

b & c) The monkey stops climbing

1. The package resumes it's downward acceleration since it is heavier than the monkey
2. The monkey continues to accelerate upwards, but at a reduced rate.
3. Again do a force decomposition, here we are looking at the whole system, so the mass is both the monkey and box $\rightarrow \Sigma m = m_1 + m_2$

$$\begin{aligned}\vec{F}_{\text{net}} &= (m_1 + m_2) \cdot \vec{a} = \vec{F} - m_1 \cdot \vec{g} \\ \vec{a} &= \frac{m_2 \cdot \vec{g} - m_1 \cdot \vec{g}}{m_1 + m_2} \\ &= \frac{(15-10)\cdot 10}{10+15} = 2 \text{ m sec}^{-2}\end{aligned}$$

The box is heavier, so we accelerate in the direction of the box.

d) What is the Tension in the rope?

We know the total acceleration of the system, (the monkey and package move together at 2 m sec^{-2}). Looking at the free-body diagram for the box, (Figure 2), we solve $\vec{F} = m\vec{a}$ for the box.

$$\begin{aligned}\vec{F}_{\text{net}} &= m_2 \cdot \vec{a}_2 = F - m_2 \cdot \vec{g} \\ \vec{F} &= m_2 \cdot (\vec{a}_2 + \vec{g})\end{aligned}$$

Note that these are vectors, and are in opposite directions, so

$$\begin{aligned}\vec{F} &= m_2 \cdot (g - a_2) \\ \vec{F} &= 15 \cdot (10 - 2) \\ \vec{F} &= 120 \text{ N}\end{aligned}$$

Since there are only two forces on the box, the force of tension from the rope and the force of gravity acting downward, we must associate the force $\vec{F}$ with tension, so $\vec{T} = 120$ N upwards.

Question 4-24 Simultaneous equations for Δx, Δy, solve for $\Delta x = 9.21$ m. So he jumped just 26 cm less than the maximum possible distance. Alternatively, use the range formula, which is derived from the above,

$$\Delta x = \frac{u^2}{g} \sin 2\theta \tag{3}$$

Question 4-25 Same as above, but solving for initial velocity, $u = 43.1$ m/s.

Question 4-34 a) 21.4 m/s. b) 24.9 m/s. c) 16.3%.

Extra Problems

Question 4-27 a) 10.0s. b) 897m.

Question 4-85 a) 2.7km. b) 76 deg clockwise.

Question 4-86 a) 1,031m. b) westward.

Question 4-91 a) 2.6×10^2 m/s. b) 45s. c) increase.

Question 4-94 a) (10.1, 0.556), (12.1, 1.51), (14.3, 2.68), (16.4, 3.99), (18.5, 5.53) (km).

Question 5-88 a) 3,260 N. b) 2.7×10^3 km. c) 1.2 m/s^2.

Question 5-90 a) 1.2×10^2 m/s^2. b) 12g. c) 1.4×10^8 N. d) 4.2 years.

Question 5-96 16 N.