Tutorial Physics 1 Week 6 Solutions

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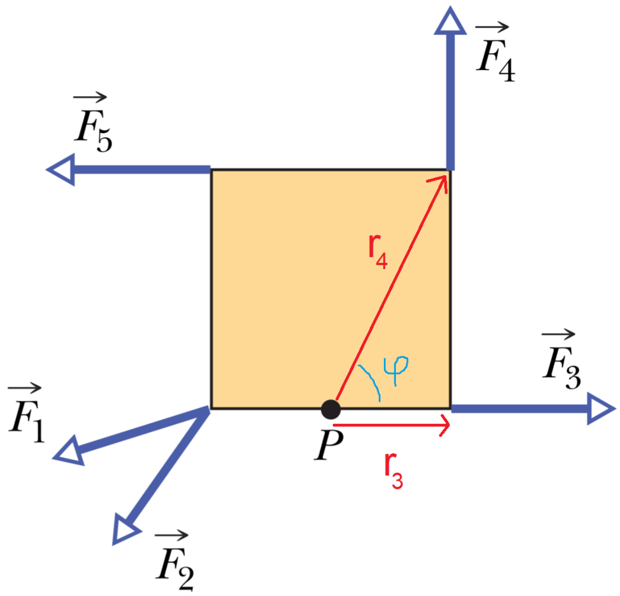
Question 10-1 The tangential velocity is the component of the velocity that is tangential to the path. For an object moving on a circular path, the velocity is *always* tangential. The acceleration of circular motion, $a = v^2/r$, acts perpendicular to the velocity, towards the centre of the circle. This acceleration is radial. If we increase the tangential velocity, then there must have been a tangential acceleration acting. The graph shows us $\omega(t)$ (omega), the angular velocity, remembering that $\vec{v} = \vec{\omega} \times \vec{r}$. Tangential acceleration increases ω. Radial acceleration is the acceleration towards the centre of the circle due to ω. a) From the graph, the magnitude of the tangential acceleration is greatest at c (largest derivative), then a, then $b = d$. b) Conversely, the radial acceleration is greatest for greatest ω, hence we have b, $a = c$, d.



Question 10-6 The magnitude of the torque, τ (tau) is given by $\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta$. Let the square have side length x. All the forces have the same magnitude, but for forces $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$, $|\vec{r}| = 1/2x$. We can immediately rank them as $0 = \vec{F_3} < \vec{F_3} < \vec{F_2}$ based on the angle between the force and moment arm ($\vec{r}$). *The moment arm is from the pivot point P to the point where the force is acting.* Similarly, we can look at $\vec{F_4}$ and $\vec{F_5}$, which have $|\vec{r}| = \sqrt{5}/2x$ (via Pythagoras). The angle between $\vec{r}_4$ and $\vec{F_4}$ is $\theta_4 = 90° + \varphi = 153°$ (draw a clear diagram). The angle between $\vec{r}_5$ and $\vec{F_5}$ is $\theta_5 = 180° - \varphi = 117°$, so $\vec{F_4} < \vec{F_5}$. $\theta_2 \sim 45°$, so $|\vec{\tau}_2| = x/2F \sin 45° = \sqrt{2}xF/4 \approx 0.354xF$. Similarly, $|\vec{\tau}_4| = \sqrt{5}/2xF \sin 153° \approx 0.51xF$. So! Finally! $0 = \vec{F_3} < \vec{F_3} < \vec{F_2} < \vec{F_4} < \vec{F_5}$!

Question 10-10 Again, torque τ (tau) is $\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta$. a) $|\vec{\tau}_1| = |\vec{\tau}_2| > |\vec{\tau}_3|$ as the torque is applied with a smaller moment arm in the third case. b) The moment of inertia depends on how the mass is distributed about the pivot point. More mass concentrated at large radii gives a larger moment of inertia, so $I_1 = I_3 > I_2$. The angular acceleration depends on the applied torque and the moment of inertia, $\tau = I\alpha \implies \alpha = \tau/I$ (does this remind you of $\vec{F} = m\vec{a}$?). Our rankings from before give $\alpha_2 > \alpha_1 > \alpha_3$. Does this make sense intuitively? Disk 2 has the equal greatest torque applied, and has the lowest moment of inertia (resistance to rotation), thus rotates most easily.

Crab Pulsar

(a) The suns diameter is $\sim 700,000$ km. The neutron star is much smaller.

  i) $f = 1/T = 1/0.033 = 30.3$ Hz, it spins 30 times per second!! (The Earth spins once per day). $\omega = 2\pi f = 190$ rad sec^{-1}.

  ii) $v = \omega r = 2\pi r/T = \frac{2\pi 10{,}000}{0.033} = 0.63c$. Quite fast!

  iii) Assume the star is a sphere, $I = \frac{2}{5}MR^2$. One solar mass is $M = 2 \times 10^{30}$ kg, and the radius is 10^4 m, $\implies I = 8 \times 10^{37}$ kg m^2. $K_{\text{rot}} = \frac{1}{2}I\omega^2 = 1.4 \times 10^{42}$ J. This is a lot!!

(b) i) $\alpha \equiv \frac{d\omega}{dt}$. We know the period increases by 1.26×10^{-5} sec year^{-1}, thus we can approximate the angular acceleration by

$$\alpha \approx \alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t}. \tag{1}$$

  With $\omega_i = 190$ rad sec^{-1} as calculated in a) i), and $\omega_f = \frac{2\pi}{T+\dot{T}}$. $\alpha = -2.3 \times 10^{-9}$ rad sec^{-2}

  ii) $\omega_f = \omega_i + \alpha t, \omega_f = 0 \implies t = 2,600$ years.

  iii) Same as above but solve for $\omega_i = 260$ rad sec^{-1} $\implies T = 0.024$ sec.

(c) Repeat part a) with your new orbital period T_0.

  i) $\omega = 260$ rad sec^{-1}.

  ii) $v = 0.87c$.

  iii) $K_{\text{rot}} = 2.7 \times 10^{42}$ J.

  i) $\Delta K_{\text{rot}} = 1.3 \times 10^{42}$ J. It has lost about half it's rotational kinetic energy!! This has mostly been deposited in the bright nebula that surrounds the pulsar, making it glow. The kinetic energy is lost as winds, streams of particles flowing away from the pulsar.

(d) You have likely assumed that the crab pulsar is a (rigid) sphere to find its moment of inertia.