Tutorial Physics 1 Week 8

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This week we will do a longer exercise, covering pretty much every aspect of simple harmonic motion. It may look long and arduous, but the questions should hopefully guide you through everything in a straightforward way. Do not input any numbers unless specifically asked to find a value.

A Fly in a Web

A fly is traveling at 2 metres per second when it gets stuck in a spider's web. The fly lands in the middle of vertical *capture thread*, which is 10cm long, and only attached to the rest of the web at both ends (top and bottom). It is immediately caught in the web, and begins oscillating in simple harmonic motion.

1 Simple Harmonic Motion

a) Write down the generic (ie no numbers involved) equation for the motion of the fly undergoing simple harmonic motion.

b) From this equation, derive the velocity of the fly as a function of time.

c) Derive the acceleration of the fly.

Simple harmonic motion occurs whenever there is a restoring force which is proportional to the displacement but with opposite sign.

i) Assume that Hooke's Law applies to the capture thread, write an equation for the force acting on the fly as it oscillates in the web.

ii) Hence or otherwise, derive and expression for the oscillation frequency of the motion.

iii) If the oscillation frequency is 20Hz, what is the spring constant in the capture thread?

d) How does the energy of the system evolve with time? Write an equation for the total energy, and each of the types of energy in the system. How do your answers change if the fly is oscillating in the horizontal direction vs the vertical direction?

e) Graph each of these energies as a function of time (horizontal motion only).

2 Damped Simple Harmonic Motion

A large golden orb weaver is alerted by the oscillating web, and steps on the thread from the top to retrieve her prize.

a) Estimate the mass of the spider, assume it is much more massive than the fly.

The spider on the web acts as a damping force, which is proportional to the velocity of the web. $F_{\text{damping}} = -bv$, where b is the damping constant.

b) Write an equation for the forces acting on the system.

c) Rewrite this as a differential equation, and verify the general solution:

$$x(t) = Ae^{-bt/2m} \cos \omega' t + \varphi$$

d) The angular frequency is now given by

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

If the new oscillation frequency is 15Hz, estimate the damping constant b.

e) How long does it take for the oscillations of the web to reach 1% of their original amplitude?

f) Graph the energy in the system as a function of time. Where does the energy go?

g) If you were the fly, how might you escape? Discuss in the content of oscillations and simple harmonic motion.

3 Pendulum

Uh oh! The **chonky** spider has been feasting on too many flies and cut corners making the web, and the web snaps at the top! Luckily, the attachment at the bottom is much stronger, so now the spider is hanging from the bottom of the thread. What is the energy in the system now? Assume that all energy due to the fly initially hitting the web has been lost. Think carefully about the scenario and draw a diagram or two.

Three possible things can happen now (or any combination of them). You will have to make some assumptions in your analysis in order to simplify the problem. Clearly state them.

a) The spider and fly can bounce up and down in simple harmonic motion.

   i) What is the equilibrium length of the thread? (assume the spring constant remains unchanged).

   ii) What is the oscillation frequency of this motion?

b) The spider and fly can start rotating with the web twisting in angular simple harmonic motion.

   i) The spider is now an instance of a torsion pendulum. If the spider completes three full rotations, estimate the torsion constant of the thread.

   ii) Hence or otherwise determine the period of this oscillation.

c) The spider and fly could swing like a pendulum, also in simple harmonic motion.

   i) Draw a diagram of the spider-pendulum at maximum extension, label all the forces acting on the system.

   ii) Consider the forces. Are they acting in the same direction? If not, create an appropriate set of axes and decompose the forces in the direction of the axes.

   iii) Consider the component of the weight force acting perpendicular to the thread. This force applies a torque to the system. Write down an expression for the torque, be careful to define a positive direction.

   iv) Using the angular version of Newton's second law, derive an expression for the angular acceleration.

   v) Hence or otherwise, derive an expression for the angular frequency of rotation and thus determine the period of oscillation of the pendulum.

   vi) Find the angular frequency and oscillation period for the system.

d) How does the moment of inertia you used in part b) differ from the moment of inertia you used in part c)?

e) Repeat part c), but replace the fly with a second spider of equal mass.